INSTITUTE OF ENGINEERING

MODEL ENTRANCE EXAM

(SET – 8)
Solutions

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date : 2081/03/29
(July 13)

Duration : 2 hours
Time : 8 A.M. – 10 A.M.

SECTION – A (**1 marks**) (1*60 = 60)

1) d
2) b
3) c
4) d
5) c
6) c
7) a
8) a
9) c
10) d
11) c
12) d
13) b
14) c
15) d $R - \mathrm{CO} - \mathrm{CH_3} \xrightarrow{\mathrm{Zn-Hg/\ conc.HCl}} \mathrm{R - CH_2 - CH_3}$
16) c 106 g of $\mathrm{Na_2CO_3}$ contains $3 \times 6.023 \times 10^{23}$ oxygen atoms.
10.6 g of $\mathrm{Na_2CO_3}$ contains $\frac{3\times6.023\times10^{23}\times10.6}{106} = 1.806 \times 10^{23}$ oxygen atoms
17) b Melting point decreases down the group because lattice enthalpy decreases with increase in size of the alkali metal. But due to covalent character of LiCl, its melting point is minimum and hence NaCl has highest melting point.
18) c $x + 4 \times 0 = 0$ $\because$ [Oxidation number of $\mathrm{CO} = 0$]
19) b $\mathrm{Na_2O}$ has antifluorite structure, coordination number of $\mathrm{Na^+}$ is 4, i.e., each $\mathrm{Na^+}$ ion is immediately surrounded by 4 $\mathrm{O^{2-}}$ ions.
20) a
21) c According to Arrhenius equation, $\mathrm{E_a}$ and temperature are related inversely. Thus, activation energy decreases with rise in temperature, thereby increasing the rate of reaction.
22) b An electronegative element has a tendency to accept electrons not to lose an electron. So, they will be having high ionization potential.
23) a During smelting process, following reaction takes place:
$Fe_2O_3 + 3C \xrightarrow{1123\,K} 2Fe + 3CO$
$Fe_2O_3 + 3CO \xrightarrow{1123\,K} 2Fe + 3CO_2$
24) a In hydroxides of alkaline earth metals, hydration energy does not alter very much while lattice energy decreases down the group due to increase in size. The overall effects of the two factor is that $(\Delta\mathrm{H_{solution}} = \Delta\mathrm{H_{lattice}} - \Delta\mathrm{H_{hydration}})$. ΔH solution of the hydroxides becomes more negative from Be to Ba and hence solubility increases from top to bottom.
25) a Mixture of $\mathrm{O_2}$ with He under pressure is supplied to sea divers for respiration.
26) c Mg, Zn → Electron
Cu, Sn → Bell metal / Bronze
Cu, Sn, Zn → Gun metal
Cu, Zn → Brass
Cu, Ni, Fe, Mn → Monel metal
Mg, Cu, Al, Mn → Duralumin
Pb, Sn, Sb → Type metal
Cu, Ni, Zn → German silver
Fe, Ni → Invar
Pb, Sn → Solder
27) d $\mathrm{F = ma = m.\frac{v}{T}}$

$[m] = [Fv^{-1}T]$

28) d in a circular motion of a particle with constant speed, the particle repeats its motion after a regular interval but does not oscillate about a fixed point. So, the motion of the particle is periodic but not simple harmonic.

29) c Relation between threshold frequency and stopping potential is given by:
$eV_0 = hv_0 - \phi$
$V_0 = \frac{h}{e}(v_0) - \frac{\phi}{e}$
Hence, the slope of the graph is $\frac{h}{e}$.

30) b Phosphorous is a pentavalent element, which donates one extra electron to the intrinsic semiconductor.

31) d $\frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 = \frac{16}{81}$

32) a B.P. increases with increase in pressure and decreases with decrease in pressure.

33) b SONAR stands for Sound Navigation and Ranging. SONAR is a device that uses ultrasonic waves to measure the distance, direction and speed of underwater objects.

34) c Chromatic aberration occurs when a lens is either unable to bring all wavelengths of colour to the same focal plane and/or wavelengths of colour are focussed at different positions in the focal plane.

35) a $i + r = 90^\circ$
$r = 90^\circ - i$

36) b Bernoulli's theorem states that when the speed of a fluid increases simultaneously the static pressure decreases or there is a decrease in the potential energy of the fluid and vice-versa. It is applicable to the fluid in an ideal state. Bernoulli's equation can be used to find the quantity which is conserved. Then, we can say, Bernoulli's theorem is a consequence of the law of conservation of that quantity.

37) a Stress required to double the length is called Young's modulus.
$\therefore Y = \frac{F}{A}$
$F = Y \times A = 2 \times 10^{11} \times 10^{-4} = 2 \times 10^7\ N$

38) d $I = \frac{E}{R+r}$
$0.2(10 + r) = 2.1$
$r = 0.5\ \Omega$

39) c The electric field intensity due to the long sheet of charge is given by:
$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$
The electric field intensity between plates $= E_1 - E_2 = 0$

40) b $B = \frac{\mu_0}{2r}$

41) b $\frac{\cos x}{\sin x}$, $\sin x \neq 0$
If $\sin x = 0$, then $x = n\pi$
But, $\sin x \neq 0$, i.e., $R = \{n\pi, n \in 1\}$

42) b $y = \sqrt{9 - x^2}$
For $f(x)$ to exist
$9 - x^2 \geq 0$
$x^2 \leq 9$
$x \leq \pm 3$
$\therefore -3 \leq x \leq 3$
Domain $= [-3, 3]$

43) c $8sec^2x - 6\sec x + 1 = 0$
$4\sec x\,(2\sec x - 1) - 1(2\sec x - 1) = 0$
$(2\sec x - 1)(4\sec x - 1) = 0$
Either, Or,
$\sec x = \frac{1}{2}$ $\sec x = \frac{1}{4}$
Value of $\sec x$ should be ≥ 1, both the values are not possible.

44) d $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} = \lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{x+1}.\left(1+\frac{5}{x+1}\right)^3 = \lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}.5}.\left(1+\frac{5}{x+1}\right)^3$
$= e^5(1)^3 = e^5$

45) a Put $\log x = t$
$\frac{1}{x}dx = dt$
$I = \int \cos t\,dt = \sin t + c = \sin(\log x) + c$

46) b We have,
$f(x) = \log|x| = \begin{cases} \log x & ,x > 0 \\ \log(-x), & x < 0 \end{cases}$
$\therefore f(x) = \begin{cases} \frac{1}{x}, & x > 0 \\ -\frac{1}{x}\times(-1) = \frac{1}{x} & ,x < 0 \end{cases}$
$f'(x) = \frac{1}{x}$ for all $x \neq 0$.

47) d

48) b It is given that $|\vec{u}| = |\vec{v}| = 1$ and θ is the acute angle between $\vec{u}$ and $\vec{v}$.
$\therefore |\vec{u}\times\vec{v}| = \sin\theta$
Now, $2\vec{u}\times 3\vec{v}$ will be a unit vector if
$|2\vec{u}\times 3\vec{v}| = 1$
$6|\vec{u}\times\vec{v}| = 1$
$6\sin\theta = 1$
$\sin\theta = \frac{1}{6}$
As θ is an acute angle, there is only one value of θ for which $2\vec{u}\times 3\vec{v}$ is a unit vector.

49) c dc's of x-axis = $(1, 0, 0)$

50) d

51) b $n = 36$
Total 8 : {(2, 6), (6, 2), (5, 3), (3, 5), (4, 4)}
$P(E) = \frac{5}{36}$

52) a $\frac{C}{5} = \frac{F-32}{9}$
$C = \frac{5}{9}(F - 32)$
$\sigma_C = \frac{5}{9}\sigma_F$
$5 = \frac{5}{9}\sigma_F$
$\sigma_F = 9$
$\Rightarrow V_F = 81$

53) b $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \frac{e^x + e^{-x}}{2}$

54) b We have,
$z_1 + z_2 + z_3 = \sqrt{2} + i$
$|z_1 + z_2 + z_3|^2 = 3$
$|z_1|^2 + |z_2|^2 + |z_3|^2 + 2\,Re(z_1\overline{z_2} + z_2\overline{z_3} + z_3\overline{z_1}) = 3$

$Re(z_1\overline{z_2} + z_2\overline{z_3} + z_3\overline{z_1}) = 0$

$z_1\overline{z_2} + z_2\overline{z_3} + z_3\overline{z_1}$ is purely imaginary.

55) c $\lim_{x\to 2}\frac{e^{3x-6}-1}{\sin(2-x)} = \lim_{x\to 2}\frac{e^{3x-6}-1}{3(x-2)} \times \frac{-3(2-x)}{\sin(2-x)} = 1 \times -3 = -3$

56) b $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{10t^9}{8t^7} = \frac{5}{4}t^2$

57) a $\int_1^2 e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx = \left[\frac{e^x}{x}\right]_1^2 = \frac{e^2}{2} - e = e\left(\frac{e}{2} - 1\right)$

58) a The line passes through the point, so zero.

59) c The differences of distances of any point on hyperbola from foci = length of transverse axis

$2 \times 4 = 8$

60) b The d.r. of line normal to the plane a, b, c and d of x-axis are 1, 0, 0.

So, angle $= \sin^{-1}\left(\frac{a.1+b.0+c.0}{\sqrt{a^2+b^2+c^2}}\right) = \sin^{-1}\left(\frac{a}{\sqrt{a^2+b^2+c^2}}\right)$

SECTION – B (2 marks) (2*40=80)

61) c

62) a

63) a

64) c

65) d $Cr(24) = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^5, 4s^1$

$l = 1$ means p-subshell and $l = 2$ means d-subshell.

$l = 1$ is for $2p^6$ and $3p^6$.

∴ Total electrons = 12

$l = 2$ is for $3d^5$.

∴ Total electrons = 5.

66) c Meq of NaOH = Meq of oxalic acid

$0.1 \times V = 20 \times 0.05 \times 2$

$V = 20$ mL

67) a $m_A = \frac{100}{2}$ kg/molecule

$m_B = \frac{64}{2}$ kg/molecule

According to Graham's law of diffusion,

$\frac{r_A}{r_B} = \sqrt{\frac{M_B}{M_A}} = \sqrt{\frac{m_B/N_0}{m_A/N_0}}$

$\frac{12\times 10^{-3}}{r_B} = \sqrt{\frac{64/2}{100/2}}$

$r_B = 15 \times 10^{-3}$

68) d When HA is 50% ionized, $[HA] = [A^-]$

$pH = pK_a + \log\frac{[A^-]}{[HA]}$

$pH = pK_a$

$pH = 4.5$

$pOH = 14 - pH = 14 - 4.5 = 9.5$

69) b The catalytic oxidation of ammonia (NH_3) typically produces nitric oxide (NO), which then reacts further to form nitrogen dioxide (NO_2). Nitrogen dioxide is then used in the preparation of nitric acid (HNO_3).

70) b As the size of the halogen atom increases from F to I, $H-X$ bond length in HX molecules also increases from H-F to H-I. The increase in $H-X$ bond length decreases the strengh of $H-X$ bond from H-F to H-I. Due to successive decrease in the strength of the $H-X$ bond from H-F to H-I, thermal stability of HX molecules also decreases from HF to HI.

71) b $CH_2=CH-C\equiv CH$

It has $4\ C-H\ \sigma-$bonds, $3\ C-C\ \sigma-$bonds and $3\ C-C\ \pi-$bonds.

72) c $CH_3CH_2COOH \xrightarrow{SOCl_2} CH_3CH_2COCl \xrightarrow{NH_3} CH_3CH_2CONH_2 \xrightarrow{Br_2+4KOH} CH_3CH_2NH_2$

This reaction is Hoffmann bromamide degradation reaction.

73) d Here, $X^{240} \rightarrow Y^{120} + Z^{120}$

Mass number of reactant $= 240$

B.E. per nucleon of reactant $= 7.6\ MeV$

Mass number of products $= 120$

B.E. per nucleon of product $= 8.5\ MeV$

Total gain in B.E. = B.E. of products – B.E. of reactants

$(120 \times 8.5 \times 2) - (240 \times 7.6) = 216\ MeV$

74) b Let apparent height of bird flying above water be x.

$\frac{1}{\mu} = \frac{\text{Real depth}}{\text{Apparent depth}}$

$\frac{1}{(4/3)} = \frac{30}{x}$

$x = 30 \times \frac{4}{3} = 40\ cm$

Apparent height of bird to the fish $= 40 + 20 = 60\ cm$

75) c $\beta = \frac{\lambda D}{d}$

$\beta \propto \lambda$

$\lambda_w = \frac{\lambda_{air}}{\mu_w} = \frac{\lambda}{4/3} = \frac{3\lambda}{4}$

$\frac{\beta_1}{\beta_w} = \frac{\lambda_1}{\lambda_w}$

$\beta_w = 0.3$ mm

76) d $f' = \frac{1}{1+\frac{v_s}{v}}.f$

$v_s = \frac{v}{2}$

So, $f' = \frac{1}{1+\frac{\frac{v}{2}}{v}}.f$

$f' = \frac{2f}{3}$

$\Delta f = f - \frac{2f}{3} = \frac{f}{3}$

77) c Torque acting on the magnet, $\tau = MB\sin 60^\circ$

Work done, $W = MB(1-\cos 60^\circ)$

$\frac{\tau}{W} = \frac{MB\sin 60^\circ}{MB(1-\cos 60^\circ)} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$\tau = \sqrt{3}W$

78) b Since the capacitors are connected in series, the charge flowing through them will be same.

Energy stored in first capacitor of capacitance 0.3 μF is:

$E_1 = \frac{Q^2}{2C_1} = \frac{Q^2}{2\times 0.3}$

Energy stored in second capacitor of capacitance 0.6 μF is:

$E_2 = \frac{Q^2}{2C_2} = \frac{Q^2}{2\times 0.6}$

$\frac{E_1}{E_2} = \frac{\frac{Q^2}{2\times0.3}}{\frac{Q^2}{2\times0.6}} = \frac{2}{1}$

79) a The voltage across all the three components are equal, so the impedance will be the same.

$R = X_L$

Current, $I = \frac{V}{\sqrt{R^2+X_L{}^2}} = \frac{10}{R\sqrt{2}}$

The potential drop across the inductor is given as

$V_L = IX_L = IR = \frac{10}{\sqrt{2}}\ V$

80) d Let u be the initial velocity that we have to find and a be the uniform acceleration of the particle. For $t = 3s$, distance travelled $S = 12m$ and for $t' = 3 + 3 = 6s$, distance travelled $S' = 12 + 30 = 42$ m

$S = ut + \frac{1}{2}at^2 \Rightarrow 12 = u \times 3 + \frac{1}{2} \times a \times 3^2$

$24 = 6u + 9a$ --- (i)

$S' = ut' + \frac{1}{2}at'^2 \Rightarrow 42 = u \times 6 + \frac{1}{2} \times a \times 6^2$

$42 = 6u + 18a$ --- (ii)

On solving, $u = 1m/s$

81) b According to law of conservation of energy,

Total energy of the body on the surface of earth = Total energy of the body at a height 'h' reached by it

$(\text{P.E.}+\text{K.E.})_{\text{surface}} = (\text{P.E.}+\text{K.E.})_{\text{maximum height}}$

$-\frac{GMm}{R} + \frac{1}{2}m\left(\frac{v_e}{2}\right)^2 = -\frac{GMm}{R+h} + \frac{1}{2}m(0)^2$ --- (i)

We know that, $v_e = \sqrt{\frac{2GM}{r}}$ --- (ii)

Substituting (ii) in (i) and solving, we get, $h = \frac{R}{3}$

82) a Mass of water pumped, $\text{m} = \text{V} \times \text{d} = 30 \times 10^3 = 3 \times 10^4$ kg

Power consumed $= \frac{\text{W}}{\text{t}} = \frac{\text{mgh}}{\text{t}} = \frac{3\times10^4\times10\times40}{900} = 13070$ Watt $= \frac{13070}{1000}$ kW $= 13.07$ kW

83) c $V = 1000v$

$\frac{4}{3}\pi R^3 = 1000\left(\frac{4}{3}\pi r^3\right)$

$R^3 = 10^3(10^{-7})^3$

$R = 10^{-6}$

$\Delta A = [4\pi R^2 - 1000(4\pi r^2)]$

$= -36\pi \times 10^{-12}\ m^2$

$U = |T\Delta A| = 7 \times 10^{-2} \times 36\pi \times 10^{-12} = 7.9 \times 10^{-12}$ J

84) b Let T_0 be the initial time period.

$T_0 = 2\pi\sqrt{\frac{l_0}{g}}$

Now, $T_1 = 2\pi\sqrt{\frac{l_0(1+\alpha t)}{g}}$

$T_1 - T_0 = T_0\left(\frac{\alpha t}{2}\right)$

This much time is lost in 1 second, since period T_0 is 1s.

Hence, total time lost $= \left(\frac{\alpha t}{2}\right) \times 7 \times 24 \times 60 \times 60 = 36.28\ s$

85) b Temperature inside the refrigerator, $T_1 = 9^\circ C = 282\ K$

Room Temperature, $T_2 = 36^\circ C = 309\ K$

Coefficient performance $= \frac{T_1}{T_2 - T_1} = \frac{282}{309-282} = 10.44$

86) c Sum of total numbers $= n.\bar{X} = 5 \times 18 = 90$
After one number excluded;
Sum $= 16 \times 4 = 64$
Excluded number $= 90 - 64 = 26$

87) a $r\left(\cot\frac{B}{2} + \cot\frac{C}{2}\right) = r\left(\frac{S}{r_2} + \frac{S}{r_3}\right) = \frac{\Delta}{S} . S\left(\frac{S-b}{\Delta} + \frac{S-c}{\Delta}\right) = \Delta\left(\frac{S-b+S-c}{\Delta}\right) = a$

88) a $P(A_1 \cap A_2) = P(A_1).P(A_2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{13} \times \frac{1}{13}$

89) c In the expansion of $(1 + \alpha x)^4$,
Middle term = ${}^4C_2$ $(\alpha x)^2 = 6\alpha^2 x^2$
In the expansion of $(1 - \alpha x)^6$,
Middle term = ${}^6C_3$ $(-\alpha x)^3 = -20\alpha^3 x^3$
∴ Coefficient of the middle term in $(1 + \alpha x)^4$ = Coefficient of the middle term in $(1 - \alpha x)^6$
$6\alpha^2 x^2 = -20\alpha^3 x^3$
$\alpha = 0, \alpha = -\frac{3}{10}$

90) a Parabola: $y^2 = 18x$
It is satisfied by (2, 6), i.e., $6^2 = 18 \times 2$
$\Rightarrow 36 = 36$

91) d $S_1 - S_2 = 0$
$(x^2 + y^2 + 4x) - (x^2 + y^2 + 2\lambda y) = 0$
$2x - \lambda y = 0$ --- (i)
Equation of the common chord: $2x - 3y = 0$ (ii)
Comparing (i) and (ii)
$\lambda = 3$

92) d $\frac{dy}{dx} = \frac{y+1}{x-1}$
$\frac{dy}{y+1} = \frac{dx}{x-1}$
Integrating, $log_e(y + 1) = log_e(x - 1) + log_e c$
$(y + 1) = (x - 1)c$
Putting $x = 1$ and $y = 2$, we get,
$3 = 0$ (not possible)
So, no solution.

93) d $\frac{dx}{dy} = \left(\frac{dy}{dx}\right)^{-1}$
$\frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}$
$\frac{d^2x}{dy^2} = \frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}\frac{dx}{dy}$
$\frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-2} \frac{d}{dx}\left(\frac{dy}{dx}\right).\frac{dx}{dy}$
$\frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-3}\left(\frac{d^2y}{dx^2}\right)$

94) a Put $\frac{1}{2}\cos^{-1}\frac{3}{5} = \theta$
$\cos^{-1}\frac{3}{5} = 2\theta$
$\cos 2\theta = \frac{3}{5}$
$\frac{1-tan^2\theta}{1+tan^2\theta} = \frac{3}{5}$

$\tan\theta = \pm\frac{1}{2}$

95) d $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

We have,

$\left|\vec{a} + \vec{b} + \vec{c}\right|^2 = a^2 + b^2 + c^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 1 + 1 + 1 + 0 = 3$

$\left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{3}$

96) c $\frac{\alpha+\beta}{\sqrt{\alpha\beta}} = \frac{-\frac{n}{l}}{\sqrt{\frac{n}{l}}} = -\sqrt{\frac{n}{l}}$

$\frac{\alpha}{\sqrt{\alpha\beta}} + \frac{\beta}{\sqrt{\alpha\beta}} = -\sqrt{\frac{n}{l}}$

$\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = -\sqrt{\frac{n}{l}}$

i.e., $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} = -\sqrt{\frac{n}{l}}$

97) c Eccentricity (e) = $\sqrt{2}$

Distance between the directrix $= \frac{2a}{3} = 10$

$\frac{2a}{\sqrt{2}} = 10$

$a = 5\sqrt{2}$

Distance between the foci $= 2ae = 2(5\sqrt{2}).\sqrt{2} = 20$

98) c $m_1 + m_2 = -\frac{2h}{b} = \frac{k}{3}$ and $m_1 m_2 = \frac{a}{b} = -\frac{1}{3}$

$m_1 + m_2 = 2m_1 m_2$

$\frac{k}{3} = -\frac{2}{3}$

$k = -2$

99) a $\int_0^{\frac{\pi}{2}} \frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}\,dx = \int_0^{\frac{\pi}{2}} \frac{(\sin x+\cos x)^2}{\sqrt{sin^2 x+cos^2 x+2\sin x\cos x}}\,dx = \int_0^{\frac{\pi}{2}} \frac{(\sin x+\cos x)^2}{(\sin x+\cos x)}\,dx = -[\cos x + \sin x]_0^{\frac{\pi}{2}}$

$= 2$

100) c $P_1 + \lambda P_2 = 0$

$(x + 2y + 3z - 4) + \lambda(4x + 3y + 2z + 1) = 0$ --- (i)

It passes through (0, 0, 0), i.e., $-4 + \lambda = 0$

$\lambda = 4$

From (i), $17x + 14y + 11z = 0$

❖❖❖❖ Thank You!!! ❖❖❖❖