BEATS ENGINEERING

INSTITUTE OF ENGINEERING

MODEL ENTRANCE EXAM

(SET – 7)
Solutions

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date : 2081/03/22
(July 06)

Duration : 2 hours
Time : 8 A.M. – 10 A.M.

SECTION – A (1 marks) (1*60 = 60)

1) c
2) d
3) c
4) d
5) d
6) d
7) c
8) b
9) b
10) b
11) a
12) a
13) c $H = I^2RT$

$H \propto I^2$

14) d
15) a Angular momentum, $L = \frac{nh}{2\pi}$

As $\frac{n}{2\pi}$ is just a number, thus dimensions of Planck's constant is same as that of angular momentum.

16) c Change in momentum, $p = p_f - p_i = -mv - mv = -2mv$

Momentum changes by 2mv in magnitude but kinetic energy remains same.

17) b $x = A\sin\omega t$

The particle is at mean position initially i.e., at $t = 0, x = 0$

At some instant t, the particle is at a position of half amplitude i.e., $x = \frac{A}{2}$

$\frac{A}{2} = A\sin\omega t$

$\sin\omega t = \frac{1}{2}$

$\omega t = \frac{\pi}{6}$

$\frac{2\pi}{T} \times t = \frac{\pi}{6}$

$t = \frac{T}{12}$

18) d $\sigma = \frac{Q}{A} = \frac{1.6\times10^{-19}}{2\times10^{-6}} = 8 \times 10^{-14}$

19) b An electric dipole when kept in uniform electric field experiences a torque but no force.

20) c
21) b The purpose of water proofing agent is to alter the surface of a solid such that it starts repelling water.

Initially, the surface was getting wet by water as water tends to spread on the surface, because contact angle is acute.

If the solid surface (e.g., clothes) are sprayed with waterproofing agent , then it changes the angle of contact from acute to obtuse. Hence, water tends to be repelled from the surface and beads up on the surface and does not easily penetrate the clothing.

22) d $E = NAB\omega\cos\theta$

The induced emf in the coil is maximum when $\cos\theta$ is maximum or $\theta = 0^{\circ}$.

23) b Friction forces are always parallel to the surfaces in contact, which in this case, are the wall and the cover of the book. This tells us that the friction force is either upwards or downwards. Because the tendency of the book is to fall due to gravity, the friction force must be in upward direction.

24) a

25) c

26) a

27) d Positive ion is always smaller and negative ion is always larger than the parent atom. So, the correct order of size of iodine species will be: $I^{-} > I > I^{+}$.

28) b $2RX + Ag_2O \xrightarrow{\Delta} R - O - R + 2AgX$

29) a Because of the highest electronegativity of Fluorine.

30) b CH_3COOK is a salt of strong base and weak acid. Its aqueous solution will be basic and pH value will be >7 ≈ 8.8.

Na_2CO_3 is a salt of strong base and weak acid. Its aqueous solution is also basic and pH value will be >10.

NH_4Cl is a salt of weak base and strong acid. So, its aqueous solution will be acidic and pH value will be less than 7.

$NaNO_3$ is the salt of strong acid and strong base. So, its aqueous solution is neutral and pH value will be equal to 7.

31) d Boiling point increases with increase in molecular mass. For the compounds with the same molecular mass, boiling point decreases with an increase in branching.

32) b Calgon is used for softening hard water.

33) c $x + 4(-2) = -3$

$x = -3 + 8 = +5$

34) b

35) d

36) d Normality = Basicity × Molarity = 2 × 0.3 = 0.6 N

37) c Atomic weight = N_A × mass of one atom $= 6.022 \times 10^{23} \times 1.8 \times 10^{-22} = 108$

38) b

39) b $\Delta G = \Delta H - T\Delta S$

40) a

41) c $2.2\sin x . \cos x = \sin x$

$\sin x\,(4\cos x - 1) = 0$

Either: $\sin x = 0$

$x = n\pi$

42) b $|x-1| \geq 0$

Range $= [0, \infty)$

43) d $b^2 - 4ac < 0$

$25 - 4.1.k < 0$

$4k > 25$

$k > \frac{25}{4} = 6\frac{1}{4}$

Least integer $= 7$

44) b $\lim_{x \to \frac{3\pi}{4}} \frac{(1+\tan x)}{\frac{1-tan^2 x}{1+tan^2 x}} = \lim_{x \to \frac{3\pi}{4}} \frac{(1+tan^2 x)}{1-\tan x} = \frac{1+tan^2\frac{3\pi}{4}}{1-tan^2\frac{3\pi}{4}} = 1$

45) d $f'(x) = \frac{\cos x}{|\cos x|}.(-\sin x)$
At $x = \frac{3\pi}{4}$; $f'\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}$

46) c Put $y = \tan^{-1} x \Rightarrow dy = \frac{1}{1+x^2} dx$
$I = \int e^y\, dy = e^y + c = e^{\tan^{-1} x} + c$

47) a Projection of $\vec{b}$ upon $\vec{a} = \frac{\vec{a}.\vec{b}}{|\vec{a}|} = \frac{5.2+(-3).1+1.2}{\sqrt{2^2+2^2+1}} = 3$

48) b $\sim[(\sim p) \wedge q] = \sim(\sim p) \vee \sim q = p \vee \sim q$

49) d Distance between P(x, y, z) and the point (x, y, 0) is:
$\sqrt{(x-x)^2 + (y-y)^2 + (z-0)^2} = |z|$

50) b

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

We observe that the possible values of prime numbers when two digits on the dice are added are 2, 3, 5, 7, and 11.
We observe that 2 occurs only once, 3 occurs 2 times, 5 occurs 4 times, 7 occurs 6 times and 11 occurs 2 times.
The number of favourable outcomes is the sum of occurrences of all the favourable outcomes.
So, the number of favourable outcomes =1+2+4+6+2=15
We know that the number of possible outcomes is 36. Thus, the probability of getting the sum of two numbers as prime numbers is = 15/36 = 5/12

51) b $\mathrm{A} - \mathrm{B} = \{\mathrm{x}: \mathrm{x} \in \mathrm{A} \text{ and } \mathrm{x} \notin \mathrm{B}\} = \mathrm{A} \cap \mathrm{B}^{\mathrm{C}}$

52) b From the end, $a = 86, d = -4$
$T_{19} = a + 18d = 86 - 72 = 14$

53) b

54) a Obvious

55) d The first plane can be filled up in 9 ways (no zero) and rest four in 10 ways each.
The total number of ways $= 9 \times 10 \times 10 \times 10 \times 10 = 90{,}000$

56) b Obvious

57) b Using L-Hospital's rule
$\lim_{x\to 2} \frac{xf(2)-2f(x)}{x-2} = \lim_{x\to 2}[f(2) - 2f'(x)] = 4 - 2 \times 4 = -4$

58) d $f'(x) = 2 - \sin x$
We know, $-1 \le \sin x \le 1$
$f'(x) = 2 - \sin x > 0\ \forall\ x \in R$
$\Rightarrow f(x)$ is an increasing function for all $x \in R$

59) a If the line is parallel to x-axis, then $m = 0$
$\frac{k-3}{k-4} = 0 \Rightarrow k = 3$

60) c Sum of distances of any point as ellipse from foci = Length of major axis $= 2 \times 5 = 10$.

SECTION – B (2 marks) (2*40=80)

61) c
62) d
63) d
64) b
65) d
66) b $MX_2(s) \rightleftharpoons M^{2+} + 2X^-$
Let, $[M^{2+}] = x$
$\therefore [X^-] = 2x$
$K_{sp} = [M^{2+}][X^-]^2$
$4 \times 10^{-12} = x(2x)^2$
$4 \times 10^{-12} = 4x^3$
$\therefore x = [M^{2+}] = 1 \times 10^{-4}$ M
67) b The given cell is:
$Fe \mid Fe^{2+} \parallel Fe^{3+} \mid Fe^{2+}$
$E^0 = E^0{}_{Fe^{3+}/Fe^{2+}} - E^0{}_{Fe^{2+}/Fe} = 0.771 - (-0.441) = 1.212$ V
68) c $CaC_2 \xrightarrow{H_2O} HC \equiv CH \xrightarrow{\text{hot iron tube}} C_6H_6 \xrightarrow{CH_3Cl, AlCl_3} C_6H_5CH_3$
Acetylene Benzene Toluene
69) d Normality $= \frac{\text{Gram equivalents}}{\text{Volume (in mL)}} \times 1000$
Gram equivalents of HCl $= \frac{0.1 \times 100}{1000} = 0.01$
No. of gram equivalents of metal carbonate = No. of gram equivalents of HCl
∴ Gram equivalents of metal carbonate = 0.01
$\frac{\text{Weight}}{\text{Equivalent weight}} = 0.01$
Equivalent weight $= \frac{2}{0.01} = 200$
70) d Rate = k[A]
i.e., $k = \frac{2.0 \times 10^{-5}}{0.01} = 2.0 \times 10^{-3}$
$t_{1/2} = \frac{0.693}{k} - \frac{0.693}{2.0 \times 10^{-3}} = 347$ s
71) a $MnO_2 + 2HCl \longrightarrow MnCl_2 + Cl_2 + 2H_2O$
Greenish yellow
$NH_3 + 3Cl_2 \longrightarrow NCl_3 + 3HCl$
Oxidation state of N in NH_3 is -3 and in NCl_3 is +3.
72) d $C_6H_5NO_2 \xrightarrow{Reduction} C_6H_5NH_2 \xrightarrow{CHCl_3, alc. KOH} C_6H_5NC \xrightarrow{H_2/catalyst} C_6H_5NHCH_3$
73) b As the number of undecayed nuclei decreases from 25% to 12.5% in 10 s, it shows that the half-life of the sample is 10 s.
i.e., $T_{1/2} = 10\ s$
Decay constant, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{10}$

Mean life, $\tau = \frac{1}{\lambda} = \frac{10}{0.693} = 14.43\ s$

74) b $\frac{\lambda}{\lambda_0} = \frac{\left[\frac{1}{2^2} - \frac{1}{3^2}\right]}{\left[\frac{1}{2^2} - \frac{1}{4^2}\right]} = \frac{5}{36} \times \frac{16}{3} = \frac{20}{27}$

$\lambda = (20/27)\lambda_0$

75) b As reflected light is completely polarized, therefore, $i_p = 60^\circ$

$\mu = \tan i_p = \tan 60^\circ = \sqrt{3}$

As, $\mu = \frac{c}{v}$

$v = \frac{c}{\mu} = \frac{3 \times 10^8}{\sqrt{3}} = \sqrt{3} \times 10^8$ m/s

76) a For minimum deviation, $i = e$

$r_1 = r_2 = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ$

Using Snell's law at the interface,

$1 \sin i = \sqrt{3} \sin r_1$

$\sin i = \sqrt{3} \sin 30^\circ = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

$i = 60^\circ$

77) d Here, $n = 25$ turns, r = 12 cm, B = 0.5 T

Since the coil is placed in a uniform magnetic field normal to the plane of the coil, the angle between magnetic moment and magnetic field direction is zero i.e., $\theta = 0^\circ$

$\therefore \tau = MB \sin\theta = MB \sin 0^\circ = 0$

78) a $I = \frac{E}{R+r}$

For the maximum current from the battery,

$R = 0$

i.e., $I = \frac{E}{r} = \frac{24}{0.8} = 30$ A

79) d For a thin uniformly charged spherical shell, the field points outside the shell at a distance x from the centre is:

$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{x^2}$

If the radius of the sphere is R, $Q = \sigma 4\pi R^2$

$\therefore E = \frac{1}{4\pi\varepsilon_0} \frac{\sigma 4\pi R^2}{x^2} = \frac{\sigma R^2}{\varepsilon_0 x^2}$

This is inversely proportional to square of the distance from the centre. It is as if the whole charge is concentrated at the centre.

80) c Here, Frequency of source, $f = 400$ Hz

Speed of sound, $v = 330$ m/s

Speed of source (i.e., train), $v_s = 30$ m/s

As the source is moving towards stationary observer

$f' = \frac{v}{v - v_s} \times f = \frac{330}{330 - 30} \times 400 = 440$ Hz

81) d According to an ideal gas equation,

$PV = nRT$

$V = \frac{nRT}{P}$

$V = \frac{nrT^2}{P}$ $\quad \because P = \frac{a}{T}$ (Given)

$dV = \frac{2nRT}{a} dT$

Work done by the gas, $dW = P\, dV$

$W = \int_T^{4T} \frac{a}{T} \frac{2nRT}{a} dT = [2nRT]_T^{4T} = 8nRT - 2nRT = 6nRT$

82) c Breaking force = Breaking stress × Area of cross section

For a given material of the wire, breaking stress is constant.

$\therefore \frac{F_1}{A_1} = \frac{F_2}{A_2}$

$F_2 = F_1 \left(\frac{A_1/4}{A_1}\right) = \frac{F_1}{4} = \frac{W}{4}$ $\quad [\because A = \pi r^2]$

83) c Acceleration of the solid sphere when it rolls without slipping down an inclined plane is

$a = \frac{g \sin\theta}{1+\frac{I}{MR^2}}$

For a solid sphere, $I = \frac{2}{5} MR^2$

$a = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5}{7}\ g \sin\theta$

Acceleration of the same sphere when it slides without friction down an same inclined plane is

$a' = g\sin\theta$

So, $\frac{a'}{a} = \frac{\frac{5}{7} g\sin\theta}{g\sin\theta} = \frac{7}{5}$

$a' = \frac{7}{5} a$

84) d Since ranges are same so shell have been fired at complementary angles. Let, they be θ and $90^\circ - \theta$.

$t_1 = \frac{2u\sin\theta}{g}$ and $t_2 = \frac{2u\sin(90^\circ-\theta)}{g} = \frac{2u\cos\theta}{g}$

$t_1 t_2 = \frac{2}{g} \cdot \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2}{g} \cdot \frac{u^2\sin 2\theta}{g} = \frac{2R}{g}$ $\quad \left[\because R = \frac{u^2 \sin 2\theta}{g}\right]$

85) c Heat lost by the water = Heat gained by the ice melting

$\mathrm{m_{water}} \times \mathrm{c_{water}} \times \Delta\mathrm{T_{water}} = \mathrm{m_{ice}} \times \mathrm{L_f}$

$300 \times 1 \times 50 = \mathrm{m_{ice}} \times 80$

$\mathrm{m_{ice}} = 187.5\ \mathrm{g}$

86) d $\frac{1}{\sqrt{A_1}} + \frac{1}{\sqrt{A_2}} + \frac{1}{\sqrt{A_3}} = \frac{1}{\sqrt{\pi r_1{}^2}} + \frac{1}{\sqrt{\pi r_2{}^2}} + \frac{1}{\sqrt{\pi r_3{}^2}} = \frac{1}{\sqrt{\pi}}\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{r}\right) = \frac{1}{\sqrt{\pi r^2}} = \frac{1}{\sqrt{A}}$

87) c $b^2 = ac$

$2log_m b = log_m a + log_m c$

$\frac{2}{log_b m} = \frac{1}{log_a m} + \frac{1}{log_c m}$

$log_a m, log_b m, log_c m$ are in H.P.

88) c Sum 7 : {(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)} = 6 ways

Sum 11 : {(5, 6), (6, 5)} = 2 ways

Favorable cases = 6 + 2 = 8

Total cases = 6 × 6 = 36

$P(E) = \frac{8}{36} = \frac{2}{9}$

89) b $(1+x)^{50}(1-x+x^2)^{50} = (1+x^3)^{50}$

$= {}^{50}C_0 + {}^{50}C_1\, x^3 + {}^{50}C_2\, (x^3)^2 + \ldots + {}^{50}C_{10}\, (x^3)^{10} + \ldots$

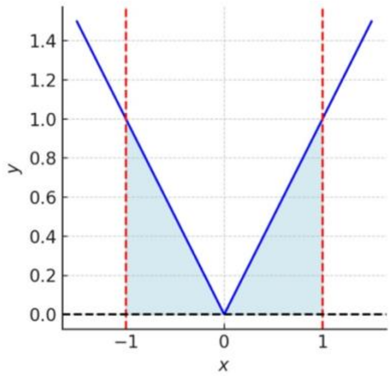
Coeff. of $(x^3)^{30} = {}^{50}C_{10}$

90) a $S_1 - S_2 = 0$
$(x^2 + y^2 - 12) - (x^2 + y^2 - 4x + 3y - 2) = 0$
$4x - 3y - 10 = 0$
Length of the perpendicular from the centre (0, 0) to the line $4x - 3y - 10 = 0$ is:
$\left|\frac{0-0-10}{\sqrt{4^2+(-3)^2}}\right| = 2$ units
Length of common chord $= 2\sqrt{12-4} = 4\sqrt{2}$ units

91) b $\frac{dy}{dx} = e^{x+y} = e^x . e^y$
$\int e^x\, dx = \int \frac{dy}{e^y}$
$e^x = -e^{-y} + c$
$e^x + e^{-y} = c$

92) a $\frac{dy}{dx} = \text{a} \cos mx \,.\, m - b \sin mx \,.\, m = m(a \cos mx - b \sin mx)$
$\frac{d^2y}{dx^2} = m^2(-a \sin mx - b \cos mx) = -m^2 y$

93) c



Area (A) $= 2\left(\frac{1}{2}.1.1\right) = 1$ sq. units

94) d $|\vec{a} + \vec{b}| = 1$
Squaring: $a^2 + 2\vec{a}.\vec{b} + b^2 = 1$
$2\vec{a}.\vec{b} = -1$
Now, $|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b})^2 = a^2 + b^2 - 2\vec{a}.\vec{b} = 1 + 1 - 2(-1) = 3$
$|\vec{a} - \vec{b}| = \sqrt{3}$

95) b $4\alpha^2 + p\alpha - 12 = 0$ --- (i)
$\frac{4\alpha^2+3p\alpha-4=0}{\alpha(p-3p)-8=0} \Rightarrow \alpha = -\frac{4}{p}$
From (i), we get, $p = \pm 2$

96) b The given equation represents an ellipse if:
$(r-2) > 0$ and $(5-r) > 0$
$r > 2$ and $r > 5$
i.e., $2 < r < 5$

97) c S.D. $= \sqrt{\frac{\Sigma(X-\bar{X})^2}{n}} = \sqrt{\frac{10}{5}} = \sqrt{2}$

98) d $f(-1) = (-1)^2 - 2 = -1$
$f(-3) = 2(-3) + 3 = -3$
i.e., $f(-1) = \frac{1}{3} f(-3)$

99) b $\frac{\frac{1}{(n+1)}}{1}+\frac{\left(\frac{1}{n+1}\right)^2}{2}+\cdots+\infty=-\left[-\frac{\left(\frac{1}{n+1}\right)}{1}-\frac{\left(\frac{1}{n+1}\right)^2}{2}-\frac{\left(\frac{1}{n+1}\right)^3}{3}-\cdots+\infty\right]$

$=-\log\left(1-\frac{1}{n+1}\right)=\log\left(\frac{n}{n+1}\right)^{-1}=\log\left(\frac{n+1}{n}\right)=\log\left(1+\frac{1}{n}\right)=\frac{1}{n}-\frac{1}{2n^2}+\frac{1}{3n^3}-\cdots+\infty$

100) c $\int_0^{\pi/4}\frac{1-\sin x}{1+\sin x}dx=(\tan x-\sec x)_0^{\pi/4}=\left(1-\sqrt{2}\right)-(0-1)=2-\sqrt{2}$

❖❖❖❖ Thank You!!! ❖❖❖❖