INSTITUTE OF ENGINEERING

MODEL ENTRANCE EXAM

(SET – 1)
Solutions

Instructions:

There are 100 multiple-choice questions, each having four choices of which only one choice is correct.

Date : 2081/02/12
(May 25)

Duration : 2 hours
Time : 8 A.M. – 10 A.M.

1) b
2) d
3) d
4) a
5) a
6) a
7) d
8) a
9) d
10) a
11) c
12) b
13) c Na_2S_2
2(+1)+2(x)=0
x = -1
14) a
15) b Order of stability of carbocations is: 3˚ > 2˚ > 1˚.
16) a Number of molecules of SO_2 in 22.4 L SO_2 at NTP = 6.02×10^{23} molecules
Number of molecules in 5.6 L SO_2 at NTP = $\frac{6.022\times10^{23}}{22.4} \times 5.6 = 1.5 \times 10^{23}$ molecules
17) d Maximum number of electrons in a shell = $2n^2$.
18) c
19) d H_2O does not contain any π-bond.
20) d $\frac{r_{CH_4}}{r_x} = \sqrt{\frac{M_x}{M_{CH_4}}}$
$2 = \sqrt{\frac{M_x}{16}}$
$M_x = 16 \times 4 = 64$
21) c
22) b All aldehydes (aliphatic and aromatic) and ketones (aliphatic methyl ketones) gives sodium bisulphite test.
23) c Order of basicity:
$2^\circ > 1^\circ > 3^\circ > NH_3$
24) a
25) c At cathode,
$2\ H_2O + 2e^- \rightarrow H_2 + 2OH^-$
$Na^+ + OH^- \rightarrow NaOH$
At anode,
$Br^- \rightarrow Br + e^-$
$Br + Br \rightarrow Br_2$
26) b
27) c Quark combination of proton = uud
Quark combination of neutron = udd
Quark combination of antineutron = $\bar{u}\bar{d}\bar{d}$
Baryon : formed by 3 quarks
The baryon number of each quark = $\frac{1}{3}$
Meson : Formed by one quark and one anti-quark

28) a The quantity of electricity is charge.
$q = it$
$[q] = [M^0L^0TA]$

29) b

30) b

31) c Due to low density, clouds have very small terminal velocity so they fall slowly and appear to be floating.

32) b With change of temperature, volume and density changes in reverse direction but mass (i.e., product of volume and density) remains unchanged. So, 50 g (given mass) weighs equal in summer and in winter.

33) b $C = \sqrt{\frac{3PV}{M}} \propto \sqrt{P}$ (since M and V be constant)
So, $\frac{C}{C_0} = \sqrt{\frac{4}{1}}$
$\Rightarrow C = 2C_0$

34) b $\lambda_{medium} = \frac{\lambda_{vacuum}}{\mu} = \frac{6000}{2} Å = 3000\ Å$

35) a

36) b $f = \frac{v}{4L} = \frac{340}{4 \times 0.25} = 340$ Hz

37) d

38) a $F = qvB \sin 0^\circ = 0$

39) c For wattless circuit, phase difference between current and voltage should be $\pi/2$. Hence, resistance R should be zero as $\cos\phi = \frac{R}{Z} = 0$.

40) b

41) b As given, sum of roots $= -3$
$-\left(\frac{2a+3}{a+1}\right) = -3$
$2a + 3 = 3a + 3$
$a = 0$
Product of roots $= \frac{3a+4}{a+1} = \frac{3(0)+4}{0+1} = 4$

42) c $1 + \frac{(\log x)^2}{2!} + \frac{(\log x)^2}{4!} + \cdots = \frac{e^{\log x} + e^{-\log x}}{2} = \frac{x + x^{-1}}{2}$

43) a $S_n = \frac{lr - a}{r - 1}$
$255 = \frac{2(128) - a}{2 - 1}$
$a = 1$

44) b Here, $n(S) = 49$
Favorable numbers are $11, 21, 31, 41$
$\therefore$ Required probability $= \frac{4}{49}$

45) c A square matrix A of order $n \times n$ is called a/an:
- Singular matrix if $|A| = 0$
- Non-singular matrix if $|A| \neq 0$
- Symmetric matrix if $A^T = A$
- Skew symmetric matrix if $A^T = -A$

46) b There are two alternatives for the button of each bulb 'on' and 'off'. To enlight the hall at least one bulb button should be 'on'.

Total no. of ways $= 2^{10} - 1 = 1023$

47) d The period of $\cos 4x$ is $\frac{\pi}{2}$ and that of $\tan 3x$ is $\frac{\pi}{3}$.

Period of $f(x) = \frac{\pi}{2} + \frac{\pi}{3} = \pi$

48) b $\lim_{x\to\infty} \frac{\tan x}{x} = \lim_{x\to\infty} \left(\frac{\sin x}{x}\right) \times \left(\frac{1}{\cos x}\right) = 0 \times (a\ finite\ number) = 0$

49) d $y = \tan^{-1}(\cot x) + \cot^{-1}(\tan x) = \frac{\pi}{2} - \cot^{-1}(\cot x) + \frac{\pi}{2} - \tan^{-1}(\tan x) = \pi - 2x$

$\frac{dy}{dx} = -2$

50) a $\theta = 0^\circ$

$\tan\theta = \frac{dy}{dx}$

$\frac{dy}{dx} = 0$

51) b $\int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin^2 x} = \int_{\pi/6}^{\pi/2} \csc x . \cot x\, dx = |-\csc x|_{\pi/6}^{\pi/2} = -\csc\frac{\pi}{2} + \csc\frac{\pi}{6} = -1 + 2 = 1$

52) a The equation of line which passes through the point (h, k) and cuts off equal intercepts on the axes is:

$x + y = h + k$

$x + y = -2 + 5$

$x + y - 3 = 0$

53) b $\tan\pi = \frac{2\sqrt{3^2 - a\times 7}}{a+7}$

$0 = \frac{2\sqrt{9-7a}}{a+7}$

$a = \frac{9}{7}$

54) b Since, the parabola passes through (3, 2)

$2^2 = 4a \times 3$

$4a = \frac{4}{3}$

Length of latus rectum $= 4a = \frac{4}{3}$

55) b For rectangular hyperbola,

Coeff. of x^2 + Coeff. of $y^2 = 0$

$5 + \lambda = 0$

$\lambda = -5$

56) a Let the parallel plane be $2x - 3y + z + k = 0$

Since, it passes though (1, -1, 2),

$2 + 3 + 2 + k = 0$

$k = -7$

Hence, the required plane is $2x - 3y + z = 7$

57) a $\tan(180^\circ + \theta) . \tan(90^\circ - \theta) = \tan\theta . \cot\theta = 1$

58) d $\sin^2\theta + 3\cos\theta = 3$

$1 - \cos^2\theta + 3\cos\theta = 3$

$\cos^2\theta - 3\cos\theta + 2 = 0$

$(\cos\theta - 1)(\cos\theta - 2) = 0$
$\cos\theta = 1, \cos\theta = 2$ (not possible)
$\theta = 0$ in $[-\pi, \pi]$
Thus, there is only one solution.

59) c Let, $\sin^{-1} x = \theta$
$\sin\theta = x$
$\therefore \cos 2\theta = \frac{1}{9}$
$1 - 2\sin^2\theta = \frac{1}{9}$
$2\sin^2\theta = \frac{8}{9}$
$x^2 = \frac{4}{9}$
$x = \pm\frac{2}{3}$

60) b $\vec{b} = -4\vec{a}$
$\therefore \vec{a} \,||\, \vec{b}$

61) d The second sentence of paragraph 1 states that probes record responses. Paragraph 2 says that electrodes accumulate much data.

62) c The tone throughout the passage suggests the potential for microprobes. They can be permanently implanted, they have advantages over electrodes, they are promising candidates for neural prostheses, they will have great accuracy, and they are flexible.

63) d According to the third paragraph, people who lack biochemicals could receive doses via prostheses. However, there is no suggestion that removing biochemicals would be viable.

64) a The first sentence of the third paragraph says that microprobes have channels that open the way for delivery of drugs. Studying the brain (choice d) is not the initial function of channels, though it is one of the uses of the probes themselves.

65) d $CH_3 - CH_2 - CH_3 \xrightarrow{Cl_2, hv} CH_3 - CH(Cl) - CH_3 \xrightarrow{Na/\ dry\ ether} CH_3 - CH(CH_3) - CH(CH_3) - CH_3$

66) c $K_a \propto Acidic\ strength$, i.e., greater the value of K_a, greater is the acidic strength and acidic strength increases with increase in -I effect.

67) a $2KOH + CO_2 \rightarrow K_2CO_3 + H_2O$
2(39+16+1) 22.4 L
=102 g
22.4 dm^3 of CO_2 at STP requires 112 g KOH
11.2 dm^3 of CO_2 at STP will require $\frac{112}{22.4} \times 11.2 = 56\ g$ KOH

68) a Equivalent weight of dibasic acid = $\frac{\text{Molecular weight}}{2} = 100$
Strength = 0.1N, mass(m) = ?, V = 100 mL
Normality (N) = $\frac{\text{mass}}{\text{E}} \times \frac{1000}{\text{V(L)}}$
M = $\frac{\text{ENV}}{1000} = \frac{100\times100\times0.1}{1000} = 1\text{g}$

69) d $CaF_2 \rightleftharpoons Ca^{2+} + 2F^-$
2×10^{-4}M 2×10^{-4}M $2\times2\times10^{-4}$M
K_{sp} of $CaF_2 = [Ca^{2+}][F^-]^2$

$= [2 \times 10^{-4}][4 \times 10^{-4}]^2$
$= 32 \times 10^{-12}(\text{mol/L})^2$

70) c At NTP, 1 mole of H_2 = 2 g
22400 mL of H_2 = 2 g
112 mL of H_2 = $\frac{2}{22400} \times 112 = 0.01$ g
1 F of electricity displace 1 g of H_2
0.01 g of hydrogen is displaced by 0.01 F of electricity
1 F can deposit 108 g of Ag
0.01 F can deposit 0.01×108 = 1.08 g of Ag

71) c According to Fajan's rule, larger the size of anion, greater is the covalent character.
Order of size of anions : $I^- > Br^- > Cl^- > F^-$
∴ Covalent character : MI > MBr > MCl > MF

72) b $2\,Mg + N_2 \xrightarrow{\Delta} Mg_3N_2 \xrightarrow{6H_2O} 3\,Mg(OH)_2 + 2NH_3$
$CuSO_4 + 4NH_3 \rightarrow [Cu(NH_3)_4]SO_4$
deep blue colour

73) d $b^2 \sin 2C + c^2 \sin 2B = 4R^2 sin^2 B.2\sin C.\cos C + 4R^2 sin^2 C.2\sin B.\cos B$
$= 8R^2 \sin B\,.\sin C(\sin B\ cos\ C + \cos B \sin C)$
$= 8R^2 \sin B\,.\sin C\,.\sin(B + C)$
$= 8R^2 \sin Bsin\ Csin\ A$
$= 8R^2 \times \frac{b}{2R} \times \frac{c}{2R} \times \frac{a}{2R} = \frac{abc}{R} = 4\Delta$

74) b $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \sin^{-1}\left(\frac{2b}{1+b^2}\right) = \tan^{-1} x$
$2\tan^{-1} a + 2\tan^{-1} b = 2\tan^{-1} x$
$2\tan^{-1}\left(\frac{a+b}{1-ab}\right) = 2\tan^{-1} x$
$x = \frac{a+b}{1-ab}$

75) c Let x^{-17} occurs in T^{p+1}
$p = \frac{15(4)-(-17)}{4+3} = 11$
$\therefore r = p + 1 = 11 + 1 = 12$

76) a $\bar{X} = \frac{\sum x_i}{n}$
$\sum x_i = 36 \times 50 = 1800$
Now, $\sum x_i - (42 + 30) = 1800 - 72 = 1728$
∴ New mean = $\frac{1728}{48} = 36$

77) d $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega+\omega^2 & \omega & -\omega^2 \end{vmatrix} = \begin{vmatrix} -\omega^2 & \omega^2 & -\omega \\ -\omega & \omega & -\omega^2 \\ -1 & \omega & -\omega^2 \end{vmatrix}$ $\because (1 + \omega + \omega^2 = 0)$
Operate $R_3 \rightarrow R_3 - R_2$
$= \begin{vmatrix} -\omega^2 & \omega^2 & -\omega \\ -\omega & \omega & -\omega^2 \\ \omega-1 & 0 & 0 \end{vmatrix} = (\omega - 1)(-\omega^4 + \omega^2) = (\omega - 1)(-\omega + \omega^2) = \omega(\omega - 1)^2$
$= \omega(\omega^2 - 2\omega + 1) = \omega^3 - 2\omega^2 + \omega = (1 + \omega) - 2\omega^2 = -\omega^2 - 2\omega^2 = -3\omega^2$

78) a f(x) is real if $\frac{\pi^2}{9} - x^2 \geq 0$

$x^2 \le \frac{\pi^2}{9}$

$|x| \le \frac{\pi}{3}$

Minimum value of $f(x) = 0$ if $x = \frac{\pi}{3}$

Maximum value of $f(x) = \tan\frac{\pi}{3} = \sqrt{3}$ if $x = 0$

$\therefore R_f = [\,0, \sqrt{3}\,]$

79) c $\lim\limits_{x\to 1} \frac{ab^x - a^x b}{x-1} \quad \left(\frac{0}{0}\ form\right)$

$= \lim\limits_{x\to 1} \frac{a(b^x.\log b) - (a^x.\log a)b}{x-1} \qquad (By\ L - Hospital's\ rule)$

$= ab\log b - ab\log a = ab\ (\log b - \log a) = ab\log\left(\frac{b}{a}\right)$

80) d $\sin y = x\sin(a+y)$

$x = \frac{\sin y}{\sin(a+y)}$

$\frac{dx}{dy} = \frac{\sin(a+y).\cos y - \sin y.\cos(a+y)}{\sin^2(a+y)} = \frac{\sin(a+y-y)}{\sin^2(a+y)}$

$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

81) c $x^2 = 32y \Rightarrow \frac{dy}{dx} = \frac{x}{16}$

$y^2 = 4x \Rightarrow \frac{dy}{dx} = \frac{2}{y}$

At (16, 8), $\left(\frac{dy}{dx}\right)_1 = m_1 = \frac{16}{16} = 1$

$\left(\frac{dy}{dx}\right)_2 = m_2 = \frac{2}{8} = \frac{1}{4}$

Required angle $= \tan^{-1}\left(\frac{1-\frac{1}{4}}{1+1.1/4}\right) = \tan^{-1}\left(\frac{3}{5}\right)$

82) d $\int \frac{dx}{\sqrt{x}(3+x)}$

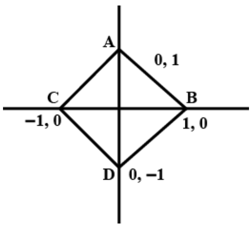
Put $\sqrt{x} = z$

$\frac{1}{2\sqrt{x}}dx = dz$

$dx = 2z\,dz$

$\int \frac{dx}{\sqrt{x}(3+x)} = \int \frac{2z\,dz}{z(z^2+3)} = 2\int \frac{dz}{z^2+3} = \frac{2}{\sqrt{3}}\tan^{-1}\frac{z}{\sqrt{3}} + c = \frac{2}{\sqrt{3}}\tan^{-1}\sqrt{\frac{x}{3}} + c$

83) d



$|x| + |y| = 1$ represent a square with length of each diagonal = 2

$\therefore A = \frac{1}{2} \times 2 \times 2 = 2$

84) a Given equation can be written as:
$3x^2 + 2hxy + (-3)y^2 + 2(-20)x + 2(25)y - 75 = 0$
It will represent a pair of straight lines if
$3(-3)(-75) + 2(15)(-20) \times h - 3(15)^2 + 3(-20)^2 + 75h^2 = 0$
$675 - 600h - 675 + 1200 + 75h^2 = 0$
$h^2 - 8h + 16 = 0$
$(h-4)^2 = 0$
$h = 4, 4$

85) b $C_1 : x^2 + y^2 + 4x + 22y + c = 0$
$C_2 : x^2 + y^2 - 2x + 8y - d = 0$
Centre $(C_2) = (1, -4)$
Since, C_1 bisects the circumference of C_2, the common chord to the circles must be the diameter of second circle.
Equation of common chord is:
$C_1 - C_2 = 0$
i.e.., $6x + 14y + (c + d) = 0$
The centre of second circle (1, -4) lies on it.
$6(1) + 14(-4) + c + d = 0$
$c + d = 50$

86) c As given, $\frac{2b^2}{a} = \frac{1}{2}(2b)$
$2b = a$
$4b^2 = a^2$
$4a^2(1 - e^2) = a^2$
$1 - e^2 = \frac{1}{4}$
$e = \frac{\sqrt{3}}{2}$

87) b As given, $cos^2\frac{\alpha}{2} + cos^2\frac{\beta}{2} + cos^2\frac{\gamma}{2} = 1$
$2cos^2\frac{\alpha}{2} + 2cos^2\frac{\beta}{2} + 2cos^2\frac{\gamma}{2} = 2$
$(1 + \cos\alpha) + (1 + \cos\beta) + (1 + \cos\gamma) = 2$
$\cos\alpha + \cos\beta + \cos\gamma = -1$

88) c Time taken by the bomb to fall through a height of 490 m is:
$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times490}{9.8}} = 10$
Distance at which the bomb strikes the ground = horizontal velocity × time
$= 360 \text{ km/hr} \times 10 \text{ s} = 360 \text{ km/hr} \times \frac{10}{3600} \text{h} = 1 \text{ km}$

98) b $\frac{E_{sphere}}{E_{cylinder}} = \frac{\frac{1}{2}I_s\omega_s^2}{\frac{1}{2}I_c\omega_c^2}$
Here, $I_s = \frac{2}{5}mR^2, I_c = \frac{1}{2}mR^2, \omega_c = 2\omega_s$
$\frac{E_{sphere}}{E_{cylinder}} = \frac{\frac{2}{5}mR^2\times\omega_s^2}{\frac{1}{2}mR^2\times(2\omega_s)^2} = \frac{4}{5}\times\frac{1}{4} = \frac{1}{5}$

90) c Gravitational potential on the surface of the shell is:

V = Gravitational potential due to particle (V_1) +
Gravitational potential due to shell itself (V_2)

$= -\frac{Gm}{R} + \left(-\frac{G(3m)}{R}\right) = -\frac{4Gm}{R}$

91) a Let the radius of bigger drop is R and smaller drop is r, then

$\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$

$R = 2r$

Terminal velocity, $v \propto r^2$

$\frac{v'}{v} = \left(\frac{R}{r}\right)^2 = \left(\frac{2r}{r}\right)^2 = 4$

$v' = 4 \times v = 4 \times 8 = 32 \text{ cm/s}$

92) a $V_T = V_0(1 + \gamma\Delta T)$

$\frac{V_T - V_0}{V_0} = \gamma\Delta T$

$\frac{0.24}{100} = \gamma \times 40$

$\gamma = 6 \times 10^{-5}\ °C^{-1}$

Coefficient of linear expansion,

$\alpha = \frac{\gamma}{3} = \frac{6\times10^{-5}}{3} = 2 \times 10^{-5}\ °C^{-1}$

93) d For an adiabatic process, $\frac{T^\gamma}{P^{\gamma-1}} = \text{constant}$

$\therefore \left(\frac{T_i}{T_f}\right)^\gamma = \left(\frac{P_i}{P_f}\right)^{\gamma-1}$

$P_f = P_i\left(\frac{T_f}{T_i}\right)^{\frac{\gamma}{\gamma-1}} = 2\left(\frac{927+273}{27+273}\right)^{\frac{1.4}{1.4-1}} = (2) \times (4)^{1.4/0.4} = (2) \times (2)^{7/2} = 2^8 = 256 \text{ atm}$

94) c Here, $f_A = 258 \text{ Hz}, f_B = 262 \text{ Hz}$

Let the frequency of unknown tuning fork be f.

It produces f_b beats with A and $2f_b$ with B. Therefore,

$f_A - f = f_b$ --- (1)

$f_B - f = 2f_b$ --- (2)

Subtract (1) from (2), we get,

$f_B - f_A = f_b$

$f_b = 262 - 258 = 4 \text{ Hz}$

From (1), we get,

$f = f_A - f_b = 258 - 4 = 252 \text{ Hz}$

95) a Total capacitance in the circuit is:

$C = \frac{3\times6}{3+6} + 2 = 2 + 2 = 4\ \mu F$

Energy $= \frac{1}{2}QV = \frac{1}{2}CV^2$ $(\because Q = CV)$

$= \frac{1}{2} \times 4 \times 2^2 = 8\ \mu J$

96) c Here, $r = 15 \text{ cm} = 15 \times 10^{-2} \text{ m}, N = 1500 \text{ turns}, I = 1.2 \text{ A}, \mu_r = 800$

Number of turns/length (n) $= \frac{N}{2\pi r} = \frac{3500}{2\pi\times15\times10^{-2}} = 3715.5$

$B = \mu_0\mu_r nI = 4\pi \times 10^{-7} \times 800 \times 3715.5 \times 1.2 = 4.48 \text{ T}$

97) b $R = \frac{V}{I_g} - G$

$R = \frac{2}{2\times10^{-3}} - 12 = 1000 - 12 = 988\ \Omega$

98) a $X_C = \frac{1}{2\pi fC} = \frac{1}{2\times3.14\times50\times0.1\times10^{-4}} = 3.2\times10^4\ \Omega$

$Z = \sqrt{R^2 + {X_C}^2} = \sqrt{100 + 10.28\times10^8} = 3.2\times10^4\ \Omega$

$I_{rms} = \frac{E_{rms}}{Z} = \frac{100}{3.2\times10^4} = 3.14\times10^{-3}\ A = 3.14\ mA$

99) d $\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\frac{A}{2}}$

According to question, $\delta_m = A$

$\sqrt{3} = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\frac{A}{2}}$

$\sqrt{3} = \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$

$\cos\frac{A}{2} = \frac{\sqrt{3}}{2} = \cos 30^\circ$

$\frac{A}{2} = 30^\circ$

$A = 60^\circ$

100) b Radius of n^{th} orbit:

$r_n = \frac{a_0 n^2}{Z}$

For hydrogen atom, $Z = 1, n = 1$(for ground state)

$r_1 = a_0$

For $Be^{3+}, Z = 4$

$r_2 = \frac{a_0 n^2}{4}$

According to question,

$r_1 = r_2$

$a_0 = \frac{a_0 n^2}{4}$

$n = 2$